Pinnacle
DEGREE & DIPLOMA
ENGINEERING

MUMBAI UNIVERSITY

SEMESTER – I

ENGINEERING MECHANICS

QUESTION PAPER – DEC 2018

Q.1. <u>Solve any four.</u>

a) Find the resultant of the parallel force system shown in Figure 1 and locate the same with respect to point C. (5 marks)

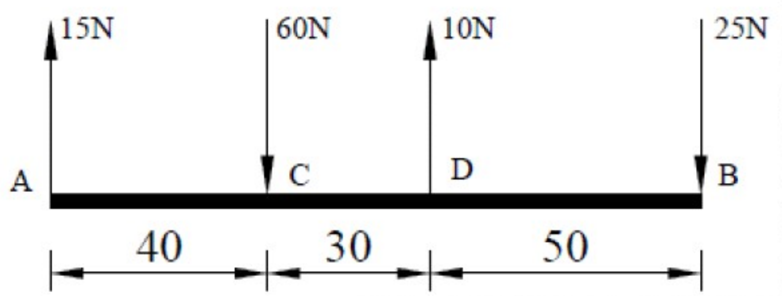


Figure 1

<u>Solution</u> :

$\sum Fx = 0$ → +ve

$\sum Fy = 15 - 60 + 10 - 25$ ↑ +ve

$= -60$ N

$R = \sqrt{Fx^2 + Fy^2}$

$= \sqrt{(-60)^2}$

$\boxed{R = 60 \text{ N}}$

$\theta = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$= \tan^{-1}(\frac{-60}{0})$

$= -90^0$

By Varignon's theorem,

$\sum Mc^F = R \times d$ +ve

d is the distance of the resultant from point C and assume R to be on the right of point C

$(15 \times 40) - (10 \times 30) + (25 \times 80) = 60 \times d$

$\boxed{d = 38.333 \text{ m}}$

The resultant force is 60 N downwards and is located 38.333 m away to the right of point C.

b) Using Instantaneous Centre of Rotation (ICR) method, find the velocity of point A for the instant shown in Figure 2. Collar B moves along the vertical rod, whereas link AB moves along the plane which is inclined at 25^0. Ɵ = 45^0 (5 marks)

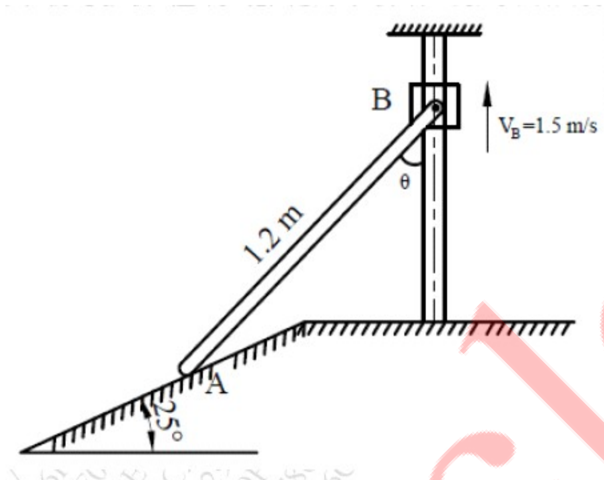
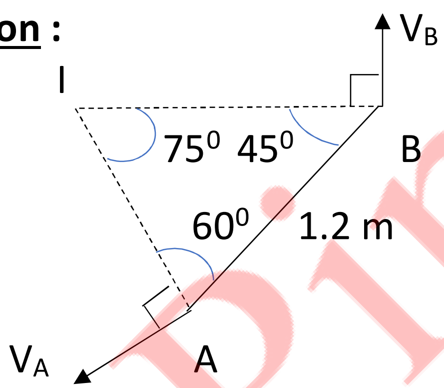


Figure 2

Solution :



By using sine rule,

$$\frac{AB}{\sin I} = \frac{BI}{\sin A} = \frac{AI}{\sin B}$$

$$BI = \frac{1.2\ X \sin(60)}{\sin(75)} = 1.076 \text{ m}$$

$$AI = \frac{1.2\ X \sin(45)}{\sin(75)} = 0.878 \text{ m}$$

ω_{AB} = BI x V_B = 1.076 x 1.5 = 1.614 rad/s

ω_{AB} = AI x V_A

$$\boxed{V_A = \frac{\omega AB}{AI} = \frac{1.614}{0.878} = 1.838 \text{ m/s}}$$

The velocity of point A for the given instance is 1.838 m/s.

c) If the support reaction at A, for the beam shown in Figure 3, is zero, then find force 'P' and the support reaction at B. (5 marks)

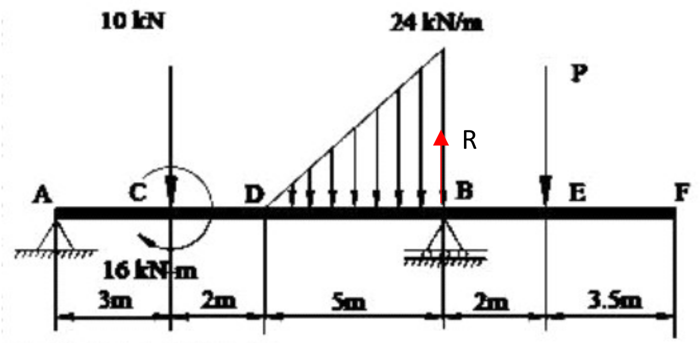
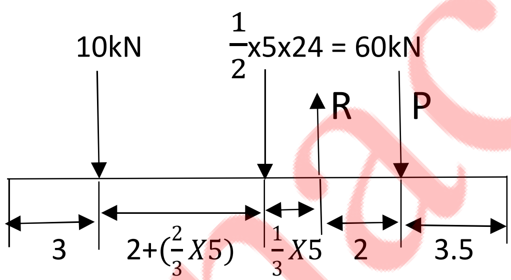


Figure 3

<u>**Solution**</u> **:**



$\sum Fx = 0$

$\sum Fy = 10 - 60 + R - P = 0$

$R - P = 50$ ……….(1)

$\sum M_B^F = (P\text{x}2) - (60\text{x}1.667) - (10\text{x}7) = 0$

$\boxed{P = 85.01\text{ kN}}$……..(2)

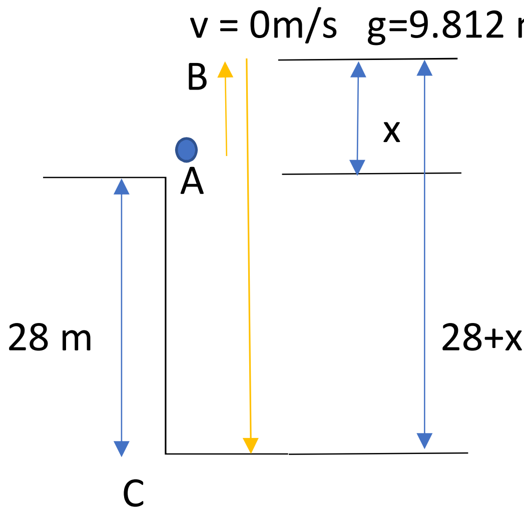
From (1) and (2)

$\boxed{R = 135.01\text{ kN}}$

The magnitudes of force P and reaction R are 85.01kN and 135.01kN respectively.

d) From the top of a tower, 28 m high, a stone is thrown vertically up with a velocity of 9m/s. After how much time will the stone reach the ground? With what velocity does it strike the ground? (5 marks)

Solution :



From A to B,

$u = 9 m/s$; $v = 0 m/s$; $a = -g = -9.812 m/s^2$

$v = u + at_1$

$0 = 9 + (-9.812 \times t_1)$

$\boxed{t_1 = 0.917 \text{ sec}}$

Now, $v^2 = u^2 + 2(-g)x$

$\boxed{x = 4.128 \text{ m}}$

From B to C,

$u = 0 m/s$ $g = 9.912 m/s^2$ $s = x + 28 = 32.128 m$

$v^2 = u^2 + 2gs$

$\boxed{v = \sqrt{2 \times 9.812 \times 32.128} = 25.11 m/s}$

$v = u + gt_2$

$t_2 = \frac{v}{g}$

$\boxed{t_2 = 2.559 \text{ sec}}$

Total time = $t_1 + t_2$

$\boxed{\text{Total time} = 0.917 + 2.559 = 3.476 \text{ sec}}$

The stone will strike the ground after 3.476 sec at a velocity of 25.11 m/s

e) For the truss shown in figure 4, find: (i) zero force members, if any (Justify your answer with FBD), (ii) support reactions at C and D. (5 marks)

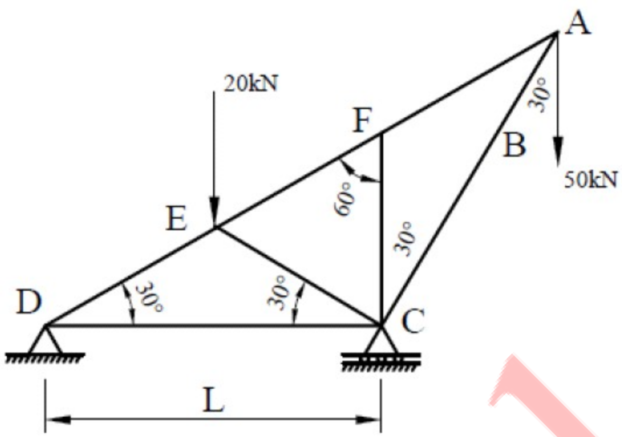
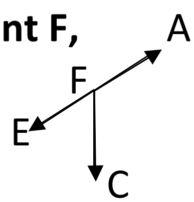


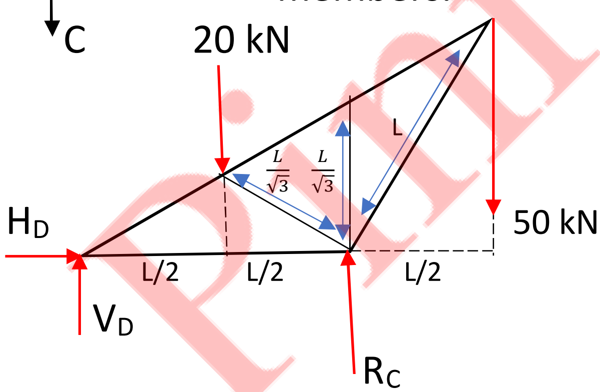
Figure 4

<u>Solution</u> :

At point F,



The member FC is a zero force member as there is no external load on F and there are two other collinear members.



$\sum F_x = 0$

$\boxed{H_D = 0}$

$\sum F_y = 0$

$-20 - 50 + R_C + V_D = 0$

$R_C + V_D = 70$ ....(1)

$\sum M_C^F = 0$

$+(V_D \times L) - (20 \times \frac{L}{2}) + (50 \times \frac{L}{2}) = 0$

$\boxed{V_D = -15 \text{ kN}}$

Substituting $V_D = -15$ kN in (1),

$\boxed{R_C = 85 \text{ kN}}$

Zero force members – FC
The magnitude of the support reactions at C and D are, $H_D = 0$ $V_D = -15$ kN and $R_C = 85$ kN respectively.

Q.2.

a) For the composite lamina shown in Figure 5, determine the coordinates of its centroid. (8 marks)

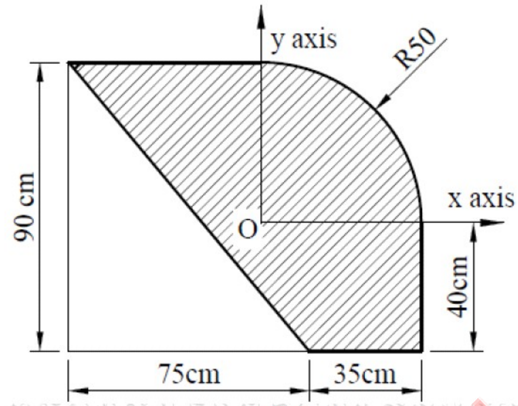


Figure 5

<u>**Solution :**</u>

Area of the shaded region = Rectangle ABFG + Rectangle OCDF + Quarter Circle OCB – Triangle AEF

Figure	Area	x coordinate	y coordinate	A_ix_i	A_iy_i
Rectangle ABFG	90 x 60 = 5400 mm^2	$\frac{-60}{2} = -30$	$50 - \frac{90}{2} = 5$	-162000	27000
Rectangle OCDF	40 x 50 = 2000 mm^2	$\frac{50}{2} = 25$	$-\frac{40}{2} = -20$	50000	-40000
Quarter Circle OCB	$\frac{1}{4}$ x π x 50^2 = 1963.495 mm^2	21.22	21.22	41665.3639	41665.3639
Triangle AEF	$-\frac{1}{2}$ x 75 x 90 = - 3375 mm^2	-35	-10	118125	33750

$\sum A_i$ = 5400 + 2000 + 1963.495 – 3375 = 5988.495

$\sum A_ix_i$ = -162000 + 50000 + 41665.3639 + 118125 = 47790.3639

$\sum A_iy_i$ = 27000 - 40000 + 41665.3639 + 33750 = 62415.3639

$$\bar{x} = \frac{\sum Aixi}{\sum Ai} = \frac{3790.3639}{5988.495} = 7.98 \text{ m}$$

$$\bar{y} = \frac{\sum Aiyi}{\sum Ai} = 10.423 \text{ m}$$

Coordinates of centroid are (7.98,10.423).

b) Replace the force system shown in Figure 6 with a single force and couple system acting at point B. (5 marks)

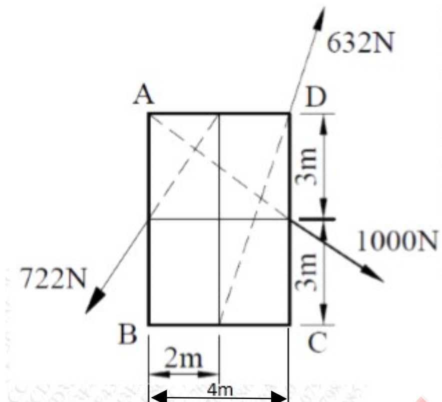
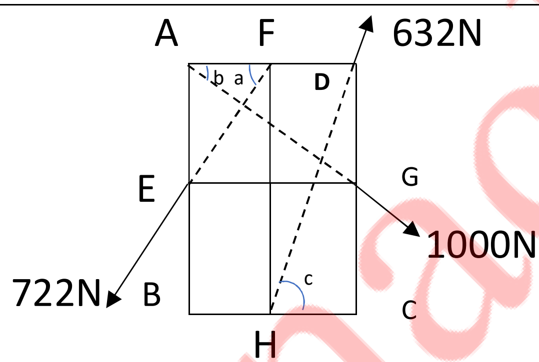


Figure 6

Solution :



In ΔAGF

b=$\tan^{-1}\frac{GD}{DA}$ = $\tan^{-1}\frac{3}{4}$ = 36.87^0

In ΔAEF

a=$\tan^{-1}\frac{AE}{AF}$ = $\tan^{-1}\frac{3}{2}$ = 56.32^0

In ΔFHD

c=$\tan^{-1}\frac{DC}{CH}$ = $\tan^{-1}\frac{6}{2}$ = 71.57^0

∑Fx = 1000cos(36.87) + 632cos(71.57) – 722cos(56.32) = 599.42

∑Fy = - 1000sin(36.87) + 632sin(71.57) – 722sin(56.32) = -601.23

R = $\sqrt{Fx^2 + Fy^2}$

= $\sqrt{599.42^2 + (-601.23)^2}$

$\boxed{R = 848.99 \text{ N}}$

Ɵ = $\tan^{-1}(\frac{\sum Fy}{\sum Fx})$

= $\tan^{-1}(\frac{-601.23}{599.42})$

= -45.086^0

$\sum M_B^F$ = - [722cos(56.32)x3] + [1000cos(36.87)x6] – [632sin(71.57)x2]

$\boxed{\sum M_B^F = 2399.66 \text{ N-m}}$

The magnitudes of the resultant force and couple at B are 848.99 N and 2399.66 N-m clockwise.

c) The link CD of the mechanism shown in Figure 7 is rotating in counterclockwise direction at an angular velocity of 5 rad/s. For the given instance, determine the angular velocity of link AB. (7 marks)

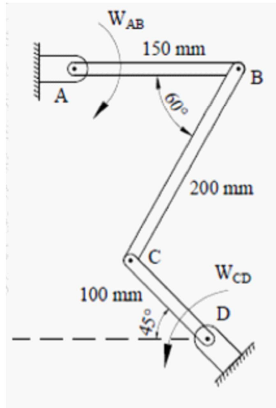


Figure 7

Solution :

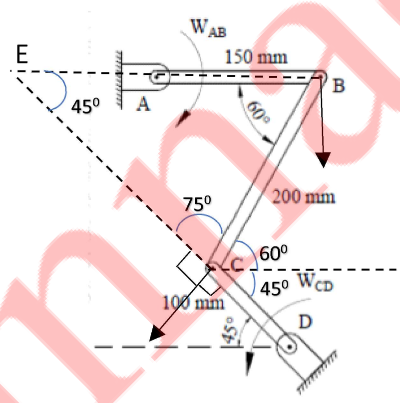


Figure 7

In ΔEBC,

Using Sine rule,

$$\frac{EB}{\sin(75)} = \frac{BC}{\sin(45)} = \frac{CE}{\sin(60)}$$

EB = $\frac{200\ X \sin(75)}{\sin(45)} = 0.27$ m   CE = $\frac{200\ X \sin(60)}{\sin(45)} = 0.24$ m

$\omega_{CD} = V_C$ x CD

$V_C = \frac{5}{0.1}$ = 50 m/s

$\omega_{BC} = V_C$ x CE = 50 x 0.24 = 12 rad/s

$\omega_{BC} = V_B$ x EB

$V_B = \frac{12}{0.27}$ = 44.44 m/s

$\boxed{\omega_{AB} = V_B \text{ x AB} = 44.44 \text{ x } 0.15 = 6.666 \text{ rad/s}}$

The angular velocity of link AB is 6.666 rad/s.

Q.3.

a) Cylinder A (diameter 1m, weight 20 kN) and cylinder B (diameter 1.5m, weight 40 kN) are arranged as shown in Figure 8. Find the reactions at all contact points. All contacts are smooth. (6 marks)

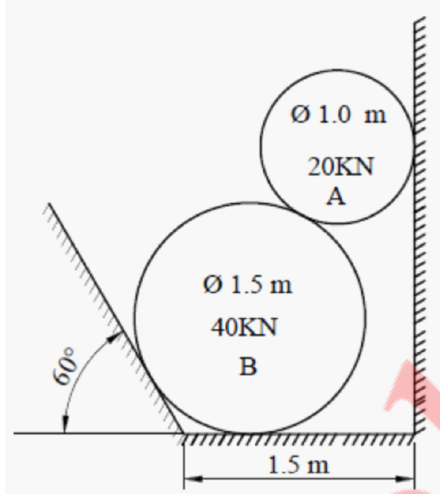


Figure 8

<u>Solution</u> :

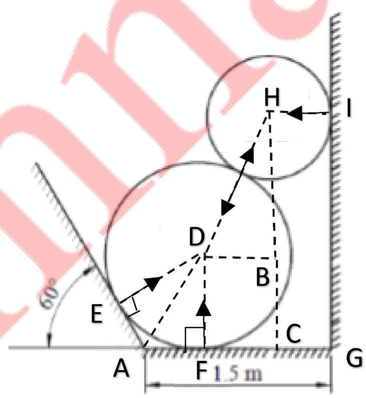


Figure 8

In ΔDAE and ΔDAF,

DA = DA;

∠DEA = ∠DFE;

By RHS rule , ΔDAE is congruent to ΔDAF

∠DAE = ∠DAF

∠DAE + ∠DAF = 2∠DAE = 180 – 60

∠DAE = ∠DAF = 60^0

In ΔDAF,

Tan(60) = $\frac{DF}{AF}$

AF = $\frac{0.75}{\tan(60)}$ = 0.433 m

AG = AF + FC + CG

1.5 = 0.433 + DB + HI

1.5 – 0.433 – HI = DB

1.5 – 0.433 – 0.5 = DB

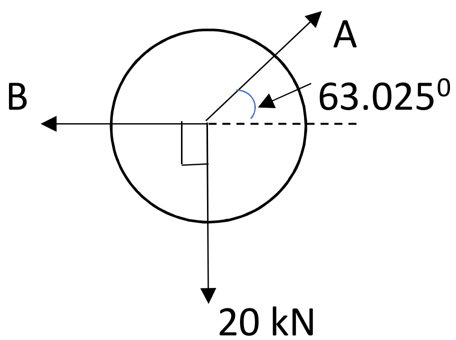
DB = 0.567 m

In ΔDBH,

Cos(D) = $\frac{DB}{DH}$ = $\frac{0.567}{0.75+\ .5}$ = 0.4536

D = $\cos^{-1} 0.4536$ = 63.025^0

Considering cylinder A,



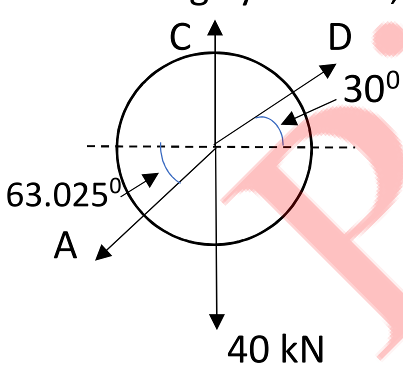
Using Lami's theorem,

$$\frac{20}{\sin(180\text{- }63.025)} = \frac{A}{\sin(90)} = \frac{B}{\sin(90+63.025)}$$

$\boxed{A = 22.44 \text{ kN}}$

$\boxed{B = 10.1795 \text{ kN}}$

Considering cylinder B,



∑Fx = 0

Dcos(30) – A cos(63.025) = 0 .....(1)

$\boxed{D = 11.75 \text{ kN}}$

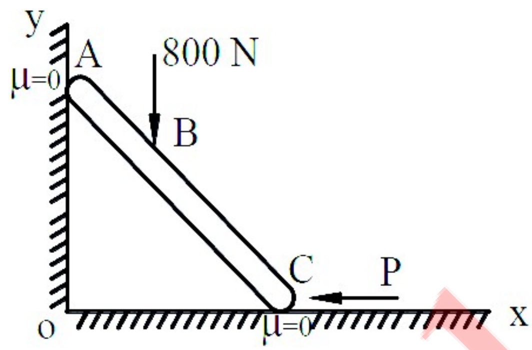
∑Fy = 0

C – 40 + Dsin(30) – Asin(63.025) = 0

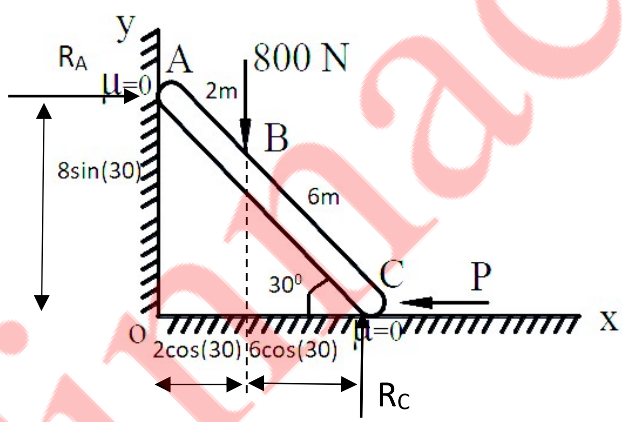
$\boxed{C = 25.876 \text{ kN}}$

The magnitudes of the reaction forces are
A = 22.44 kN
B = 10.1795 kN
C = 25.876 kN
D = 11.75 kN

b) **Using Principle of Virtual Work, determine the force P which will keep the weightless bar AB in equilibrium. Take length AB as 2m and length AC as 8m. The bar makes an angle of 30^0 with horizontal. All the surfaces in contact are smooth. Refer Figure 9.** **(6 marks)**



Solution :



Active force	Co-ordinate of point of application	Virtual displacement
-P	8cos(30)	-8sin(30)dθ
-800	6sin(30)	6cos(30)dθ

By principle of Virtual Work,

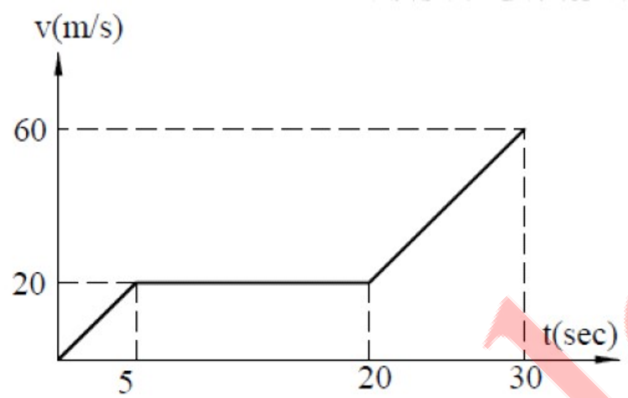
$\sum W = 0$

-P(-8sin(30)dθ)-800(6cos(30)dθ)=0

$\boxed{P = 1039.23N}$

The magnitude of the force P is 1038.23N.

c) **Velocity-time diagram for a particle travelling along a straight line is shown in Figure 10. Draw acceleration-time and displacement-time diagram for the particle. Also find important values of acceleration and displacement.**
(8 marks)



Solution :

From 0 to 5 sec, u=0

v = at

a = $\frac{v}{t}$ = $\frac{20}{5}$ = 4 m/s^2

s = $\frac{1}{2}$at^2

s = $\frac{1}{2}$ x 4 x 5^2

s = 50m

From 5 to 20 sec,

u = v = 20 m/s

a = ut

s = 20 x 15

s = 300

Total displacement after 20 sec = $S_{0\text{ to }5}$ + $S_{5\text{ to }20}$ = 300 + 50 = 350m

From 20 to 30 sec, u=20m/s v=60m/s

v = u+at

a = $\frac{v-u}{t}$ = $\frac{60-20}{10}$ = 4 m/ s^2

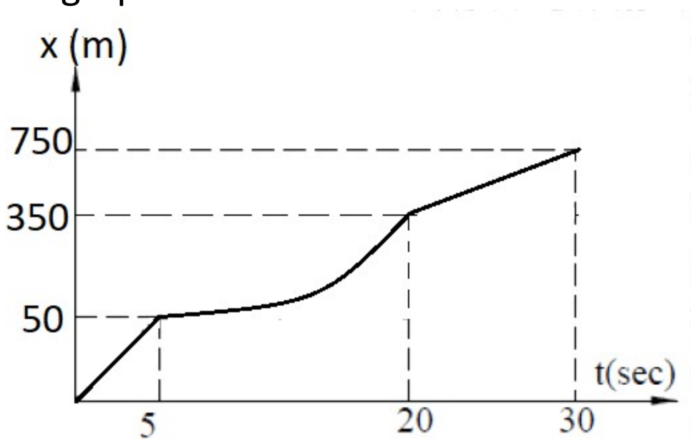
s = ut + $\frac{1}{2}$at^2

s = (20x10) + $\frac{1}{2}$ x 4 x 10^2

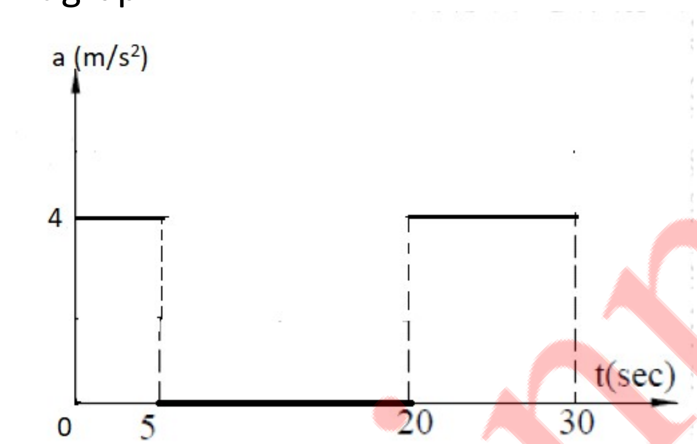
s = 400m

Total displacement after 30 sec = $S_{0 \text{ to } 20} + S_{20 \text{ to } 30}$ = 350 + 400 = 750m

x-t graph

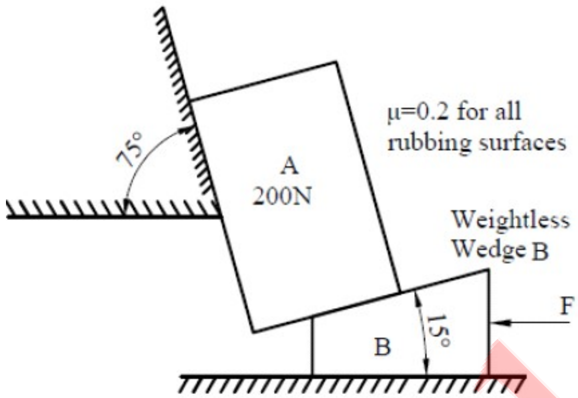


v-t graph

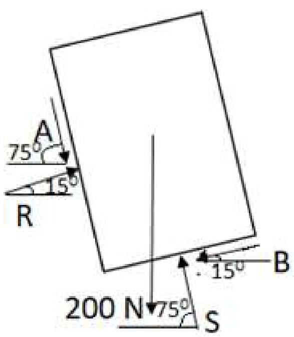


Q.4.

a) Find the force 'F' to have motion of block A impeding up the plane. Take coefficient of friction for all the surfaces in contact as 0.2. Consider the wedge B as weightless. Refer Figure 11. (7 marks)



<u>Solution</u> :



Consider block A,

$\sum Fx = 0$

$-B\cos(15) - S\cos(75) + A\cos(75) + R\cos(15) = 0$

$-0.2S\cos(15) - S\cos(75) + 0.2R\cos(75) + R\cos(15)$

$S(-0.2\cos(15)-\cos(75)) + R(0.2\cos(75)+\cos(15)) = 0$ ......(1)

$\sum Fy = 0$

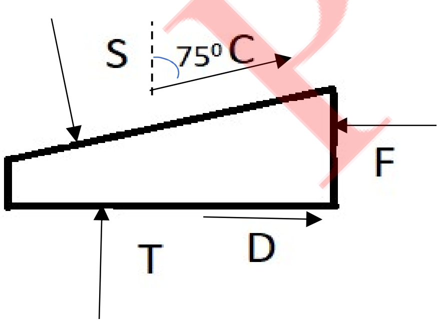
$-200 - B\sin(15) + S\sin(75) + R\sin(15) - A\sin(75) = 0$

$-0.2S\sin(15) + S\sin(75) - 0.2R\sin(75) + R\sin(15) = 200$

$S(-0.2\sin(15)+\sin(75)) + R(\sin(15)-0.2\sin(75)) = 200$ ......(2)

Now, Solving (1) and (2),

S=212.019 N R=94.168 N



Consider block wedge B,

$\sum Fx = 0$

$-F + S\cos(75) + C\sin(75) + D = 0$

$F = S\cos(75) + 0.2S\sin(75) + 0.2T$

$\sum Fy = 0$

$T - S\sin(75) + C\cos(75) = 0$

$T = S\sin(75) - 0.2S\cos(75)$

$T = 193.8197$ N

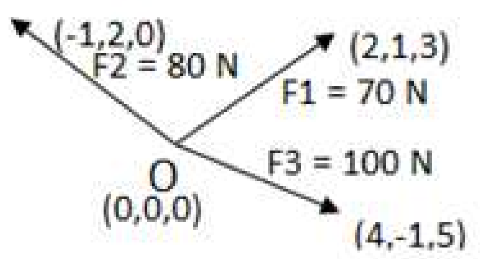
$F = S\cos(75) + 0.2S\sin(75) + 0.2TN$

$\boxed{F = 134.597 \text{ N}}$

The magnitude of the force F is 134.597 N.

b) Three forces F1, F2 and F3 act at the origin of Cartesian coordinate axes system. The force F1 (= 70N) acts along OA whereas F2 (= 80N) acts along OB and F3 (= 100N) acts along OC. The coordinates of the points A, B and C are (2,1,3), (-1,2,0) and (4,-1,5) respectively. Find the resultant of this force system. (5 marks)

<u>Solution</u> :



$\overline{F1} = 70[\frac{2i+j+3k}{\sqrt{2^2+1^2+3^2}}]$ = 37.416 i + 18.708 j + 56.12 k

$\overline{F2} = 80[\frac{-i+2j}{\sqrt{-1^2+2^2}}]$ = -35.777 i + 71.554 j

$\overline{F3} = 100[\frac{4i-j+5k}{\sqrt{4^2+-1^2+5^2}}]$ = 61.721 i - 15.43 j + 77.152 k

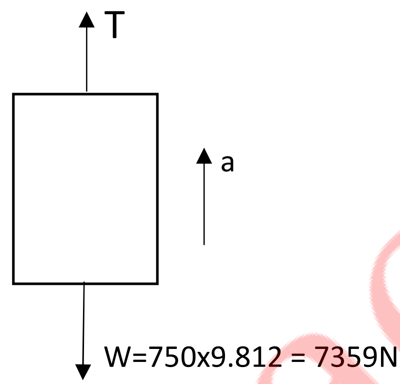
Resultant = $\overline{F}$ = $\overline{F1}$ + $\overline{F2}$ + $\overline{F3}$ = 37.416 i + 18.708 j + 56.12 k - 35.777 i + 71.554 j + 61.721 i - 15.43 j + 77.152 k

$\boxed{\text{Resultant } = 63.36\ i + 74.823\ j + 133.272\ k}$

The resultant of the force system = 63.36 i + 74.823 j + 133.272 k

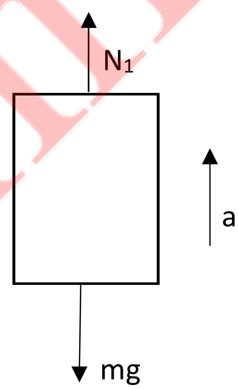
c) A 75kg person stands on a weighing scale in an elevator. 3 seconds after the motion starts from rest, the tension in the hoisting cable was found to be 8300N. Find the reading of the scale, in kg during this interval. Also find the velocity of the elevator at the end of this interval. The total mass of the elevator, including mass of the person and the weighing scale, is 750kg. If the elevator is now moving in the opposite direction, with same magnitude of acceleration, what will be the new reading of the scale? (8 marks)

<u>Solution</u> :

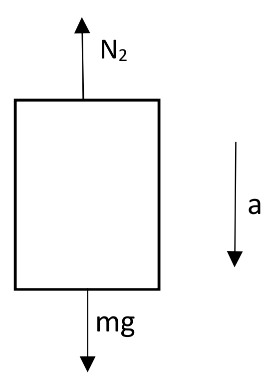


t = 3 sec
u = 0 m/s
T = 8300 N
$\sum F = ma$
$T - W = 750 \times a$
$8300 - 7359 = 750 \times a$
$a = 1.255\ m/s^2$

$v = u + at$
$v = 0 + (1.255 \times 3)$
$v = 3.765\ m/s$



For upward motion,
$N_1 - mg = ma$
$N_1 = ma + mg$
$N_1 = m(a+g)$
$N_1 = 75(1.255+9.812)$
$N_1 = 830.025\ N$
$\boxed{N_1 = 84.59\ kg}$



For downward motion,
$N_2 - mg = -\ ma$
$N_2 = mg - ma$
$N_2 = m(g-a)$
$N_2 = 75(9.812 - 1.255)$
$N_2 = 641.775\ N$
$\boxed{N_2 = 65.407\ kg}$

In upward motion the reading on the weighing scale is 84.59 kg, final velocity at the end = 3.765 m/s and the reading on the weighing scale is 65.407 kg in the downward direction.

Q.5.

a) The cylinder B, diameter 400mm and weight 5kN, is held in position as shown in Figure 12 with the help of cable AB. Find the tension in the cable and the reaction developed at contact C. (4 marks)

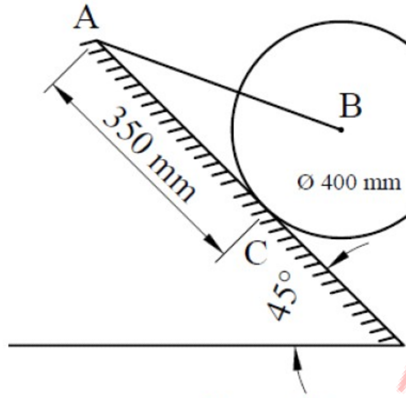


Figure 12

Solution :

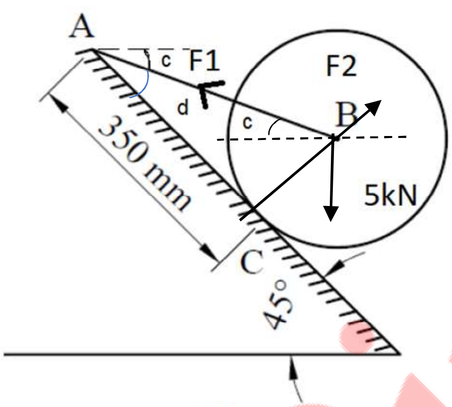


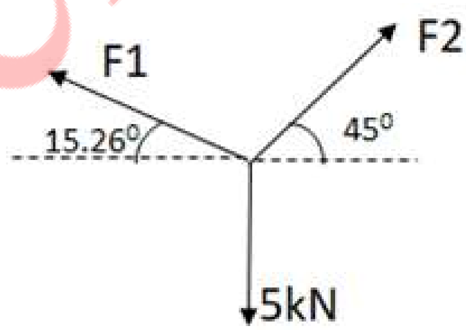
Figure 12

$\tan(d) = \frac{200}{350}$

$d = 29.74^0$

$c + d = 45^0$

$c = 15.26$



Using Lami's theorem,

$$\frac{5}{\sin(180-15.26-45)} = \frac{F1}{\sin(90+45)} = \frac{F2}{\sin(90+15.26)}$$

$\boxed{F1 = 4.072 \text{ kN}}$

$\boxed{F2 = 5.555 \text{ kN}}$

The magnitudes of the tension in the cable and the reaction developed at C are 4.072 kN and 5.555 kN.

b) Find the weight W_B so as to have its impending motion down the plane. Take weight of block A as 2kN. The pin connected rod AB is initially is in horizontal position. Refer Figure 13. Coefficient of friction = 0.25 for all surfaces. (5 marks)

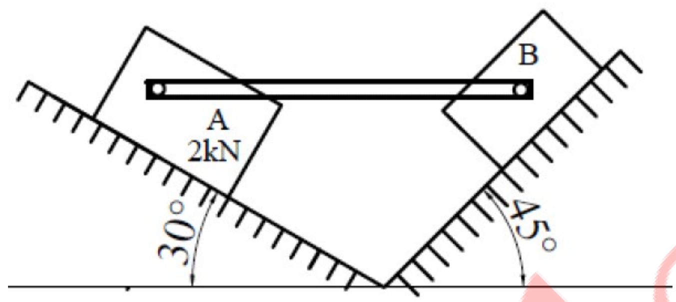
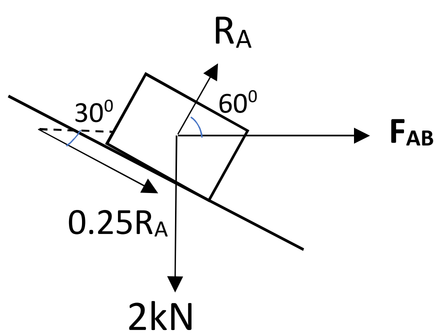
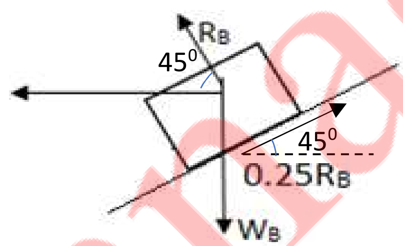


Figure 13

<u>Solution</u> :



A



B

Consider block A

$\sum Fx=0$

$F_{AB} + R_A\cos(60) + 0.25R_A\cos(30) = 0$

$F_{AB} + 0.7165R_A = 0$ ......(1)

$\sum Fy=0$

$-2 + R_A\sin(60) - 0.25R_A\sin(30) = 0$

$0.741R_A = 2$

$R_A = 2.699$ kN .....(2)

$F_{AB} = -0.7165R_A$

$F_{AB} = -1.934$ kN ......(3)

Considering block B,

$\sum Fx=0$

$-F_{AB} - R_B\cos(45) + 0.25R_B\cos(45) = 0$

$F_{AB} = -0.53R_B$

$R_B = 3.649$ kN

$\sum Fy=0$

$-W_B + R_B\sin(45) + 0.25R_B\sin(45)$

$\boxed{W_B = 3.225 \text{ kN}}$

The weight of the block B is 3.225 kN.

c) Two springs, each having stiffness of 0.6N/cm and length 20 cm are connected to a ball B of weight 50N. The initial tension developed in each spring is 1.6N. The arrangement is initially horizontal, as shown in Figure 14. If the ball is allowed to fall from rest, what will be its velocity at D, after it has fallen through a height of 15 cm? (5 marks)

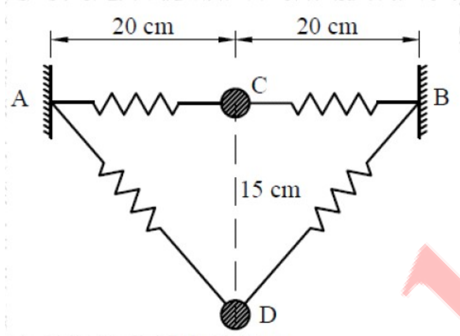


Figure 14

Solution :

Initial tension = 1.6 N

T = kx

1.6 = 0.6x

x_i = 2.667 cm ….(initial deformation)

Free length of the spring = l = 20 – x_i = 20 – 2.667 = 17.333 cm

The length of the spring at D = AD = $\sqrt{20^2 + 15^2}$ = 25 cm

Deformation at point D = x_f = 25 – 17.333 = 7.667 cm

Using work energy principle,

∑Work done = Change in K.E

Gravitational work + Spring work = $\frac{1}{2}m(V_D^2 - V_C^2)$

mgh +2 $[\frac{1}{2}k(x_i^2 - x_f^2)] = \frac{1}{2} \times 50 \times (V_D^2 - 0)$

$(50x9.812x15) + 0.6(2.667^2 - 7.667^2) = 25V_D^2$

$7359 - 31.002 = 25V_D^2$

$V_D^2 = 293.12$

$\boxed{V_D = 17.12 \text{ cm/s}}$

The velocity of the ball at point D is 17.12 cm/s.

d) Two balls, A (mass 3kg) and B (mass 4kg), are moving with velocities 25 m/s and 40 m/s respectively (Refer Figure 15). Before impact, the direction of velocity of two balls are 300 and 500 with the line joining their centers as shown in Figure 15. If coefficient of restitution for the impact is 0.78, find the magnitude and the direction of velocities of the balls after the impact. (6 marks)

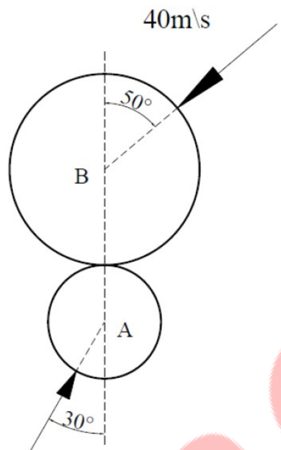
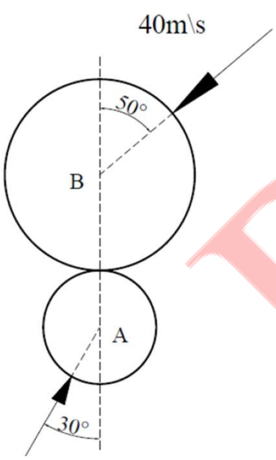


Figure 15

Solution :



u_{Ax} = 25sin(30) = 12.5 m/s

u_{Ay} = 25cos(30) = 21.65 m/s

u_{Bx} = 40sin(50) = 30.64 m/s

u_{By} = -40cos(50) = -25.71 m/s

Momentum is conserved only along the line of action.

$m_A u_{Ay} + m_B u_{By} = m_A v_{Ay} + m_B u_{By}$

$3(21.65) + 4(-25.71) = 3v_{Ay} + 4v_{By}$

$3v_{Ay} + 4v_{By} = -37.89$ .....(1)

$e = \frac{VBy - V}{UAy - UBy}$

$0.78 = \frac{VBy - VAy}{21.65 + 25.71}$

$v_{By} - v_{Ay} = 36.9408$ .....(2)

Solving (1) and (2)

$\boxed{v_{Ay} = -26.52 \text{ m/s} = 26.52 \text{ m/s}} \downarrow$

$\boxed{v_{By} = 10.42 \text{ m/s}} \uparrow$

The magnitude of the velocities and direction of A and B are 26.52 m/s downwards and 10.42 m/s upwards respectively.

Q.6

a) For the truss shown in Figure 16, find the forces in members DE, BD and CB. (5 marks)

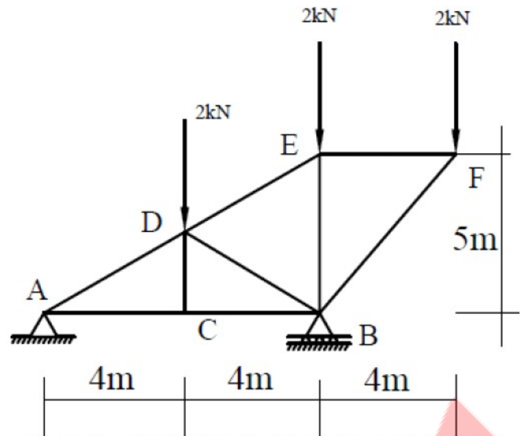
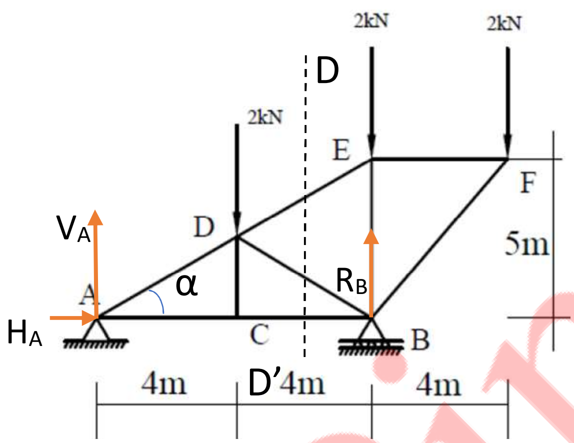


Figure 16

<u>**Solution**</u> **:**



$\sum Fx = 0$

$H_A = 0$

$\sum Fy = 0$

$V_A - 2 - 2 - 2 + R_B = 0$

$V_A + R_B = 6$ ....(1)

$\sum M_A^F = 0$

$(2x4) + (2x8) + (2x12) - (R_Bx8) = 0$

$R_B = 6$ kN

Now,

$V_A + R_B = 6$

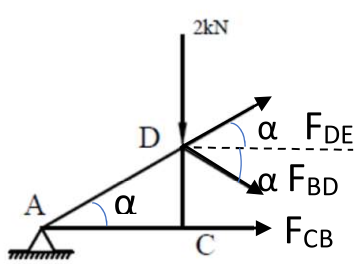
$V_A = 6 - 6 = 0$ kN

In ΔABE,

$\text{Tan}(\alpha) = \frac{EB}{AB} = \frac{5}{8}$

$\alpha = \tan^{-1} 0.625 = 32^0$

Taking section DD',



$\sum Fx = 0$

$F_{DE}\cos(32) + F_{BD}\cos(32) + F_{CB} = 0$ ......(1)

$\sum Fy = 0$

$-2 + F_{DE}\sin(32) - F_{BD}\sin(32) = 0$

$F_{DE}\sin(32) - F_{BD}\sin(32) = 2$ ....(2)

$\sum M_D^F = 0$

F_{CB} x perpendicular distance of F_{CB} from D = 0

F_{CB} = 0 kN…..(3)

Solving (1), (2) and (3),

$\boxed{F_{DE} = 1.887 \text{ kN}}$

$\boxed{F_{BD} = -\ 1.887 \text{ kN}}$

$\boxed{F_{CB} = 0 \text{ kN}}$

The forces in the members DE, BD and CB are 1.887 kN (compression), 1.887 kN (tension) and 0 kN respectively.

b) A particle moves in x-y plane with acceleration components $a_x = -3 m/s^2$ and $a_y = -16t\ m/s^2$. If its initial velocity is $V_0 = 50 m/s$ directed at 35^0 to the x–axis, compute the radius of curvature of the path at t = 2 sec. (6 marks)

<u>Solution</u> :-

At t=0

$V_0 = 50$ m/s at 35^0 to the x-axis

$V_x = 50\cos(35) = 40.96$ m/s

$V_y = 50\sin(35) = 28.68$ m/s

Given, $a_x = -3\ m/s^2$ and $a_y = -16t\ m/s^2$

Integrating, $V_x = -3t + c_1$ and $V_y = -8t^2 + c_2$

At t=0

$c_1 = 40.96$ and $c_2 = 28.68$

Now,

$V_x = -3t + 40.96$ and $V_y = -8t^2 + 28.68$

At t=2sec

$V_x = -3(2) + 40.96$ and $V_y = -8(2^2) + 28.68$

$V_x = 34.96$ m/s and $V_y = -3.32$ m/s

$a_x = -3\ m/s^2$ and $a_y = -32\ m/s^2$

V = $\sqrt{Vx^2 + Vy^2}$ = $\sqrt{34.96^2 + (-3.32)^2)}$ = 35.12 m/s

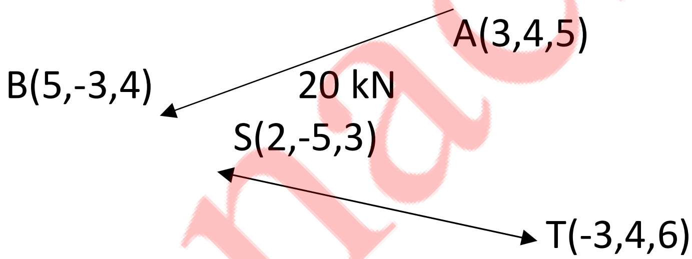
Radius of curvature at t = 2sec,

$$\boxed{R = \frac{V^3}{|Vxay - Vyax|} = \frac{35.12^3}{|(34.96\ X - 32) - (-3.32\ X - 3)|} = 38.38\ m}$$

The radius of curvature of the path at t = 2 sec is 38.38 m

c) A force of magnitude of 20kN, acts at point A(3,4,5)m and has its line of action passing through B(5,-3,4)m. Calculate the moment of this force about a line passing through points S(2,-5,3) m and T(-3,4,6)m. (5 marks)

<u>Solution</u> :



$\overline{F1} = 20\left[\frac{(5-3)i + (-3-4)j + (4-5)k}{\sqrt{(5-3)^2 + (-3-4)^2 + (4-5)^2}}\right]$ = 5.44 i – 19.05 j – 2.72 k kN

$\overline{M_S}^{F1} = \overline{SA} \times \overline{F1} = \begin{bmatrix} i & j & k \\ 3-2 & 4-(-5) & 5-3 \\ 5.44 & -19.05 & -2.72 \end{bmatrix}$ = 13.62 i + 13.6 j – 68.01 k kN-m

$|M_S^{F1}| = \sqrt{(13.62)^2 + (-13.6)^2 + (-68.01)^2}$ = 70.68 kN-m

$\widehat{ST} = \frac{\overline{ST}}{|\overline{ST}|} \frac{(-3-2)i + (4+5)j + (6-3)k}{\sqrt{(-3-2)^2 + (4+5)^2 + (6-3)^2}}$ = -0.466 i + 0.839 j + 0.28 k

Moment about the line,

$M_{ST}^{F1} = M_S^{F1} . \widehat{ST}$ = (13.62 i + 13.6 j – 68.01 k).(-0.466 i + 0.839 j + 0.28 k)

= -6.35 + 11.41 – 19.04

$$\boxed{M_{ST}^{F1} = -13.98\ \text{kN-m}}$$

Vector form,

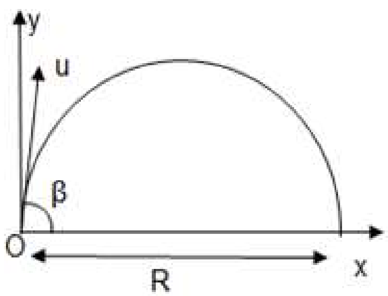
$\overline{M_{ST}}^{F1} = M_{ST}^{F1} . \widehat{ST}$ = -13.98(-0.466 i + 0.839 j + 0.28 k)

$\boxed{M_{ST}^{F1} = 6.51\ i - 11.73\ j - 3.91\ k}$

The moment of the force about a line passing through points S(2,-5,3) m and T(-3,4,6)m is -13.98 kN-m (magnitude) and 6.51 i – 11.73 j – 3.91 k (vector form).

d) Find an expression for maximum range of a particle which is projected with an initial velocity of '*u*' inclined at an angle of 'β' with the horizontal. **(4 marks)**

Solution :



Consider a particle performing projectile motion.
R – Horizontal Range
T – Total flight time

Considering vertical components of motion,

$s = ut + \frac{1}{2}at^2$

$0 = u\sin(\beta) - \frac{1}{2}gT^2$

$$T = \frac{2u\sin(\beta)}{g}$$

Considering horizontal components of motion,

$s = ut + \frac{1}{2}at^2$

$R = u\cos(\beta)T + 0$ …….(as acceleration in x direction is zero)

$R = u\cos(\beta) \times \frac{2u\sin(\beta)}{g}$

$R = \frac{u^2\sin(2\beta)}{g}$

For maximum Range, sin(2β) should be maximum, i.e. sin(2β) = 1, i.e. 2β = 90, i.e. $\beta = 45^0$

$\boxed{R_{max} = \frac{u^2}{g}}$